Reinforcement Learning

Michèle Sebag ; TP : Diviyan Kalainathan
TAO, CNRS — INRIA — Université Paris-Sud

Nov. 13th, 2017

Credit for slides: R. Sutton, F. Stulp

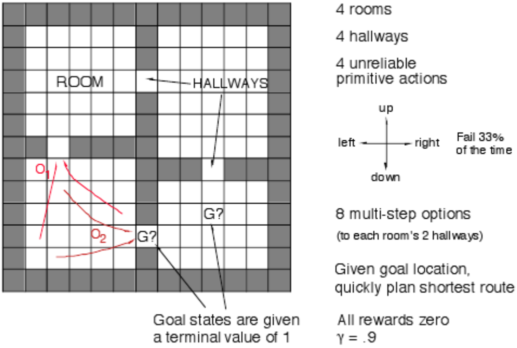
Overview

Markov Decision Process

Dynamic Programming

- Temporal differences and eligibility traces
- Q-learning
- Partial summary

Ingredients



Issues

- How does the world behave ?
- How does the agent behave ?
- What is the goal

- Markov Decision Process (S,A,p,r)
- Policy $\pi : S \mapsto A$
- Optimize expected cumulative rewards

Markov Decision Process

- State space S Terminal states $T \subset S$
- Action space A
- Transition $p(s, a, s')$: probability of arriving in s' after doing a in s
- Reward $r(s, a)$: goodies for doing a in s
  sometimes, $r(s)$: just for being in s

Markov property

Future only depends upon current state

Remark

This can always hold.
But ?

Markov Decision Process

- State space S Terminal states $T \subset S$
- Action space A
- Transition $p(s, a, s')$: probability of arriving in s' after doing a in s
- Reward $r(s, a)$: goodies for doing a in s

  sometimes, $r(s)$: just for being in s

Markov property

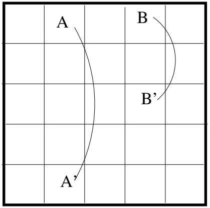
Future only depends upon current state

Remark

This can always hold.
But ? more expensive

Policy – Quality

$$\pi : S \mapsto A$$



A –> A', reward 10

B –> B', reward 5

$$s_0, a_0 = \pi(s_0), r_0, s_1, a_1 = \pi(s_1), r_1, s_2, \ldots$$

Episodic

$$R(\pi) = r(s_0) + r(s_1) + \ldots r(s_K)$$

Continuous

$$R(\pi) = \sum \gamma^{k+1} r(s_k)$$

- s_0 drawn after probability p_{Init}
- s_i drawn with probability $p(s_{i-1}, \pi(s_{i-1}, \cdot))$

Designing an RL problem

Choices

- Which state space ?
- Size of the search space
- Reward function
- How unpredictable is the environment (if multiple agents...)
- Which discount factor ?

Some problems...

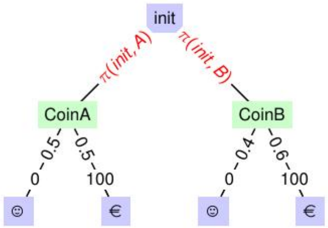
- Optimal autonomous driving (safe, fast, comfortable)
- Optimal trading on the stock-market
- Policy that optimizes your happiness during your life
- Policy that optimizes long-term hapiness of humanity

Which discount factor ?

Features of RL problems

- Finite vs. Infinite
- Discrete vs. Continuous
- Model-based vs. Model-free
- Episodic vs. Continuing
- Markovian vs. Non-Markovian
- Observable vs. Partially Observ.

The coin problem



Compute return

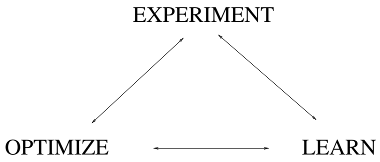
- Random policy

Which optimal policy ?

The global RL problem

3 interleaved tasks

- Learn a world model (p, r)
- Decide/select (the best) action
- Explore the world



Milestones

MDP Main Building block

General settings

	Model-based	Model-free
Finite	**Dynamic Programming**	Discrete RL
Infinite	(optimal control)	Continuous RL

Overview

Markov Decision Process

Dynamic Programming

- Temporal differences and eligibility traces
- Q-learning
- Partial summary

Algorithmic paradigms

Greedy optimization

Define incrementally a solution, based on myopic optimization of some criterion

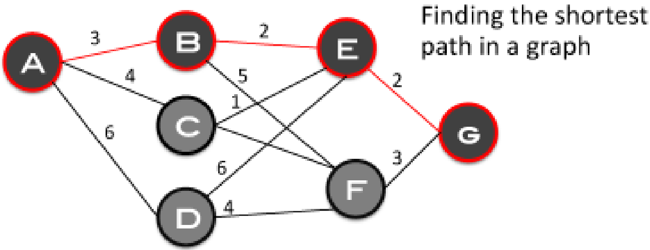
Divide and conquer

- Define subproblems
- Find optimal solutions for subproblems
- Combine solutions to subproblems

Dynamic programming

- Recursively decompose the problem in subproblems
- Bottom-up: solve small sub-problems
- Assemble their solutions to solve larger subproblems
- Can be close to brute-force search.

Dynamic programming, an example



Algorithm

recursion + memoization

- $D(N, N') = \infty$
- $D(N, N) = 0$
- $D(N, N') = c(\text{Edge}(N, N'))$ iff it exists
- Repeat until no change
  - If $D(N_1, N_2) > D(N_1, N_3) + D(N_3, N_2)$,
    $$D(N_1, N_2) = D(N_1, N_3) + D(N_3, N_2)$$

Dynamic Programming

Bellman, 50s

Context

- Computer Science: theory, AI, graphics,
- Information theory
- Control theory
- BioInformatics
- Operation research

Algorithms

- Viterbi for Hidden Markov Models
- Smith-Waterman for sequence alignment
- diff in Unix for comparing two files
- Bellman-Ford for shortest paths in graphs

Value function

Intuition

- What is the value of being in a state ?
- The value is good if this state is associated to a (delayed) reward

Caveat

- The value depends on the state
- The value depends on the policy
- $V_\pi(s)$ is the expected cumulative reward when starting in s and following π

Observation

$$\begin{aligned} R_t &= r_0 + \gamma r_1 + \ldots + \gamma^k r_k + \ldots \\ &= \sum_{k=0}^{\infty} \gamma^k r_k \end{aligned}$$

Expectation

$$V_\pi(s) = \mathbb{E}[R_t | s_0 = s]$$

Bellman equation

$$
\begin{aligned}
V_\pi(s) &= \mathbb{E}[R_t | s_0 = s] \\
&= \mathbb{E}[\textstyle\sum_{k=0}^{\infty} \gamma^k r_k | s_0 = s] \\
&= \mathbb{E}[r(s)] + \mathbb{E}(\textstyle\sum_{k=1}^{\infty} \gamma^k r_k | s_0 = s] \\
&= \mathbb{E}[r(s)] + \textstyle\sum_{s'} p(s, \pi(s), s') \mathbb{E}[\textstyle\sum_{k=1}^{\infty} \gamma^k r_k | s_1 = s'] \\
&= \mathbb{E}[r(s)] + \gamma \textstyle\sum_{s'} p(s, \pi(s), s')\, \mathbb{E}[\textstyle\sum_{k=0}^{\infty} \gamma^k r_k | s_0 = s'] \\
&= \mathbb{E}[r(s)] + \gamma \textstyle\sum_{s'} p(s, \pi(s), s')\, V(s')
\end{aligned}
$$

Bellman equation

- A theoretical property of value functions
- Optimal Bellman equation
  Define

$$V^*(s) = max_\pi V_\pi(s)$$

Then, π^* is an optimal policy if and only if

$$V_{\pi^*} = V_*$$

$$\pi^*(s) = \arg\max_a p(s, a, s')V_*(s')$$

(What is needed to compute $\pi^*(s)$ from V_* ?)

Policy evaluation

Truncate at k time steps

$$V_{\pi,k}(s) = \mathbb{E}\left[\sum_{\ell=1}^{k} \gamma^{\ell} r_\ell | s_0 = s\right]$$

$$lim_{k\to\infty} V_{\pi,k}(s) = V_\pi(s)$$

($V_{\pi,k}$ is an approximation of V_π; can we bound the approximation error ?)

Iterative policy evaluation

Given policy π

Init

$$\forall s \in S, V_\pi(s) = 0$$

Loop

$$\begin{array}{ll} \Delta = 0 & \\ \text{For each} & s \in S \\ & v = V(s) \\ & V(s) = r(s) + \gamma \sum_{s'} p(s, \pi(s), s')\, V(s') \\ & \Delta = max(\Delta, |v - V(s)|) \end{array}$$

Until $\Delta < \varepsilon$

Output $V \approx V_\pi$

Policy Improvement

Intuition

- Build $V_\pi(s)$
- You are in s
- This is the model-based setting
- Can you think of better than doing $\pi(s)$?

Policy Improvement

Intuition

- Build $V_\pi(s)$
- You are in s
- This is the model-based setting
- Can you think of better than doing $\pi(s)$?

Improved π'

$$\pi'(s) = \arg\max_a \left\{ p(s, a, s')\, V_\pi(s') \right\}$$

Algorithm

1. Define π
2. Build V_π
3. π': Policy improvement(π)
4. $\pi = \pi'$; Goto 2

This converges toward optimal π^* but takes for ever

Value Iteration

Policy evaluation recall

$$V_{\pi,k+1}(s) = r(s) + \gamma \sum_{s'} p(s, \pi(s), s')\, V_{\pi,k}(s')$$

Value iteration more greedy

Value Iteration

Policy evaluation recall

$$V_{\pi,k+1}(s) = r(s) + \gamma \sum_{s'} p(s,\pi(s),s')\, V_{\pi,k}(s')$$

Value iteration more greedy

$$V_{k+1} = r(s) + \gamma \arg\max_{a} \sum_{s'} p(s,a,s')\, V_k(s')$$

Policy evaluation vs Value iteration

	Policy evaluation	Value iteration
Init	π	V
loop	$a = \pi(s)$	$a = argmax$
Output	V_π	Greedy policy (V)

Initialization

Random ?

- Educated initialisation is better
- See Inverse Reinforcement Learning
- https://www.youtube.com/watch?v=0JL04JJjocc
- https://www.youtube.com/watch?v=VCdxqn0fcnE
- More: ICML 2004, Pieter Abbeel and Andrew Ng

Policy iteration

Principle

- Modify π step 1
- Update V until convergence step 2

Getting faster

- Don't wait until V has converged before modifying π.

Discussion

Policy and value iteration

- Must wait until the end of the episode
- Episodes might be long

Can we update V on the fly ?

- I have estimates of how long it takes to go to RER, to catch the train, to arrive at Cité-U
- Something happens on the way (bump into a friend, chat, delay, miss the train,...)
- I can update my estimates of when I'll be home...

TD(0)

1. Initialize V and π
2. Loop on episode
   2.1 Initialize s
   2.2 Repeat

$$\begin{aligned}&\text{Select action } a = \pi(s)\\&\text{Observe } s' \text{ and reward } r\\&V(s) \leftarrow V(s) + \alpha(\underbrace{r + \gamma V(s')}_{R} - V(s))\\&s \leftarrow s'\end{aligned}$$

   2.3 Until s' terminal state

Discussion

Update on the spot ?

- Might be brittle
- Instead one can consider several steps

$$R = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2})$$

Find an intermediate between

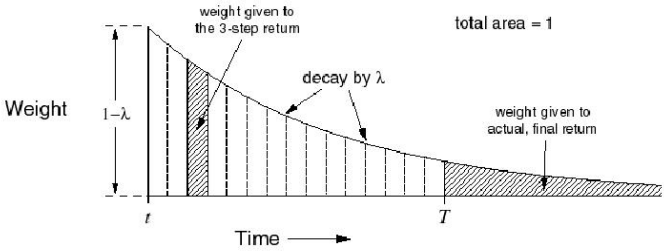
- Policy iteration

$$R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \dots$$

- TD(0)

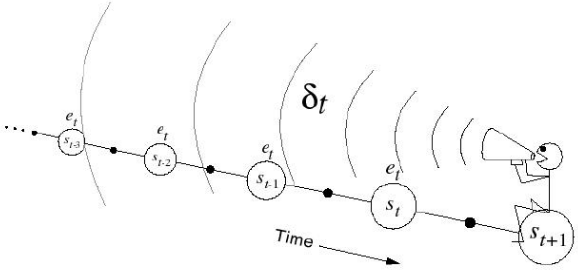
$$R_t = r_{t+1} + \gamma V_t(s_{t+1})$$

TD(λ), intuition



$$R_t^\lambda = \underbrace{(1-\lambda)\sum_{n=1}^{T-t-1}\lambda^{n-1}R_t^{(n)}}_{} + \underbrace{\lambda^{T-t-1}R_t}_{}$$

TD(λ), intuition, followed



$$\delta_t = r_{t+1} + \gamma V_t(s_{t+1}) - V_t(s_t)$$

TD(λ)

1. Initialize V and π
2. Loop on episode
   2.1 Initialize s
   2.2 Repeat

$$
\begin{array}{l}
a = \pi(s) \\
\text{Observe } s' \text{ and reward } r \\
\delta \leftarrow r + V(s') - V(s) \\
e(s) \leftarrow e(s) + 1 \\
\qquad \text{For all } s“ \\
\qquad V(s'') \leftarrow V(s“) + \alpha\delta e(s'') \\
\qquad e(s'') \leftarrow \gamma\lambda e(s'') \\
s \leftarrow s'
\end{array}
$$

   2.3 Until s' terminal state

Q-learning

Principle: Iterate

- During an episode (from initial state until reaching a final state)
- At some point explore and choose another action;
- If it improves, update $Q(s, a)$:

$$Q(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \times \left[\overbrace{\underbrace{r(s_{t+1})}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \underbrace{\max_{a_{t+1}} Q(s_{t+1}, a_{t+1})}_{\text{max future value}}}^{\text{learned value}} - \underbrace{Q(s_t, a_t)}_{\text{old value}} \right]$$

Equivalent to

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t)(1 - \alpha) + \alpha[r(s_{t+1}) + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1})]$$

Partial summary

Strengths

- Optimality guarantees (converge to global optimum)...

Weaknesses

- ...if each state is visited often, and each action is tried in each state
- Number of states: exponential wrt number of features

Discussion

Values and emotions

More: Antonio Damasio. Descartes' Error: Emotion, Reason, and the Human Brain